# Sparse Phase Retrieval Algorithms for Frequency-Domain Optical Coherence Tomography

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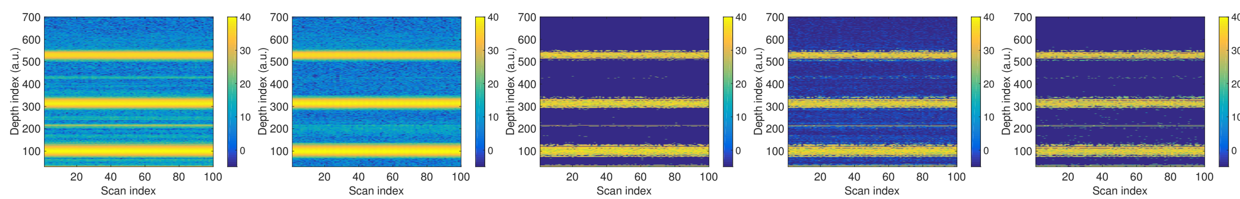
### 1. Sparse phase retrieval in frequency-domain optical coherence tomography (FDOCT)

The goal in FDOCT is to reconstruct the backscattered wave $x(z)$, which encodes information about the specimen under consideration as a function of depth $z$, from Fourier magnitude measurement $I(\omega) = S(\omega) \left|1 + \int_{-\infty}^{+\infty} x(z) e^{-\mathrm{j}\omega z} \mathrm{d}z\right|^2$ recorded by the spectrometer, where $S(\omega)$ is the source power spectrum. This is equivalent to retrieving the phase of the signal $\delta(z) + x(z)$, where $\delta(z)$ is the *Dirac delta function*, from the square of its Fourier magnitude spectrum. Since the reflected wave $x(z)$ exhibits strong peaks only at those locations where there is a significant change of refractive index in the specimen, the assumption of sparsity is apt in the context of FDOCT. In practice, one acquires samples of $I(\omega)$ and $S(\omega)$, since the measurements are acquired only at discrete wavelengths.




(a) Direct inversion (b) Homomorphic (c) max-$s$ (d) $\beta = 0.40$ (e) $\beta = 0.50$

Figure 1: State-of-the-art techniques ((a) and (b)) vis-à-vis max-$s$ (in (c)) and $\ell_0$-RAAR ((d) and (e)) for the glass specimen.

### 2. Proposed fixed-point algorithms and their performance

Let $\mathcal{S}_0 = \{\boldsymbol{u} \in \mathbb{R}^n : \boldsymbol{u} = \boldsymbol{\Psi}\boldsymbol{\alpha},\ \|\boldsymbol{\alpha}\|_0 \leqslant s\}$ be the set of all $s$-sparse signals in basis $\boldsymbol{\Psi}$; and $\mathcal{M} = \{\boldsymbol{u} \in \mathbb{R}^n : |\boldsymbol{F}\boldsymbol{u}| = \boldsymbol{y}\}$, where $\boldsymbol{F}$ denotes the $n \times n$ discrete Fourier transform (DFT) matrix, be the set of all signals whose DFT magnitude matches with the measurement vector $\boldsymbol{y}$. In sparse PR, we seek to find a signal $\hat{\boldsymbol{x}}$ such that $\hat{\boldsymbol{x}} \in \mathcal{S}_0 \cap \mathcal{M}$. We recently proposed two iterative fixed-point algorithms [1], referred to as *max-$s$* and *relaxed averaged alternating reflections with $\ell_0$ constraint* ($\ell_0$-RAAR), for solving this non-convex feasibility problem. Their update rules are given by $\boldsymbol{x}^{t+1} = \mathcal{P}_{\mathcal{S}_0}\left(\mathcal{P}_{\mathcal{M}}\boldsymbol{x}^t\right)$ and $\boldsymbol{x}^{t+1} = \mathcal{T}_\beta\left(\boldsymbol{x}^t\right)$, respectively, where $\mathcal{T}_\beta$ is a combination of appropriately defined projection and reflection operators [2]. The max-$s$ algorithm has the *error reduction property*, meaning that the measurement domain reconstruction error does not increase with iterations. The $\ell_0$-RAAR algorithm offers a generalized parameterized framework for FDOCT reconstruction, of which max-$s$ and the direct inversion technique turn out to be special cases, corresponding to $\beta = \frac{1}{2}$ and $\beta = 0$, respectively. The performance of max-$s$ and $\ell_0$-RAAR is validated on a specimen formed by stacking two microscopic glass slides. The sparsity is chosen as $s = 0.05n$, where $n = 2048$ samples are taken along each scan-line. The sparsifying basis $\boldsymbol{\Psi}$ is chosen to be the orthonormal *Haar* basis. We observe (cf. Fig. 1) that the proposed techniques yield tomograms without *autocorrelation artifacts* and superior signal-to-background noise ratio compared with the state-of-the-art.

### References

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