PARTIALLY-COHERENT IMAGING IN TERMS OF THE WIGNER DISTRIBUTION FUNCTION

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Brightfield, darkfield, phase contrast or differential interference contrast microscopes are all partially coherent imaging systems. Although image formation in fully coherent or incoherent systems has been well understood since the time of Abbe and Rayleigh, imaging in a partially coherent system was first analyzed by Hopkins only in 1953. He propagated the mutual intensity through the optical system, but did not give an expression for the mutual intensity of the image itself. Understanding the image formation process is important for image simulation and interpretation. The mutual intensity is a four dimensional (4D) quantity containing information about the modulus and phase of the image wave field, which depends on the object's complex refractive index in 3D. The mutual intensity is related to other representations in phase space, including the Wigner distribution function (WDF). These are measurable quantities, and many papers have described methods for their experimental determination.

In a recent letter we described how a model for imaging in a partially-coherent optical microscope can be developed, based on filtering of the WDF of the object amplitude transmission: the phase-space imager model [1]. In the limit of a slowly varying object, as is the case if the Rytov approximation for scattering is valid, the WDF has a simple physical interpretation. The model much simplifies to give a spatially varying intensity that depends on the local phase gradient of the sample, according to the value of the phase gradient transfer function (PGTF) of the imaging system, which gives the image intensity for a locally-constant phase gradient object. This justifies consideration of image formation in terms of the WDF of the object. The method can be applied to brightfield or phase contrast systems [2, 3].

Explicit expressions are presented for the different phase space representations of the image wave field, including the WDF, and the expressions separated into system and object dependent parts. In addition, an explicit relationship between the defocused partially coherent transmission cross-coefficient (TCC) and the WDF in the image plane is derived. The stochastic wave field in the image plane can be described in terms of a 6D system-dependent kernel, a Fourier transform of the system mutual spectrum, the region of overlap of two displaced objective pupils and the effective source. The image intensity can be expressed in terms of a 4D kernel, the PSI-kernel [3], which is the convolution in spatial frequency of the source and the WDF of the objective pupil, and is given by a marginal (projection) of the 6D WDF kernel.

1. S. B. Mehta and C. J. R. Sheppard, "An equivalent of the point spread function for partially coherent imaging," *Optica* **2**, 736-739 (2015).
2. S. B. Mehta and C. J. R. Sheppard, "Phase-space representation of partially-coherent imaging systems using the Cohen class distribution," *Opt. Lett.* **35**, 348-350 (2010).
3. S. B. Mehta and C. J. R. Sheppard, "Using the phase-space imager to analyze partially coherent imaging systems: bright-field, phase contrast, differential interference contrast, differential phase contrast, and spiral phase contrast," *J. mod. Optics* **57**, 718-739 (2010).