Sparse Phase Retrieval Algorithms for Frequency-Domain Optical Coherence Tomography

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1. Sparse phase retrieval in frequency-domain optical coherence tomography (FDOCT)

The goal in FDOCT is to reconstruct the backscattered wave \( x(z) \), which encodes information about the specimen under consideration as a function of depth \( z \), from Fourier magnitude measurement \( I(\omega) = S(\omega) \left| 1 + \int_{-\infty}^{\infty} x(z) e^{-j\omega z} dz \right|^2 \) recorded by the spectrometer, where \( S(\omega) \) is the source power spectrum. This is equivalent to retrieving the phase of the signal \( \delta(z) + x(z) \), where \( \delta(z) \) is the Dirac delta function, from the square of its Fourier magnitude spectrum. Since the reflected wave \( x(z) \) exhibits strong peaks only at those locations where there is a significant change of refractive index in the specimen, the assumption of sparsity is apt in the context of FDOCT. In practice, one acquires samples of \( I(\omega) \) and \( S(\omega) \), since the measurements are acquired only at discrete wavelengths.

2. Proposed fixed-point algorithms and their performance

Let \( S_0 = \{ u \in \mathbb{R}^n : u = \Psi \alpha, \| \alpha \|_0 \leq s \} \) be the set of all \( s \)-sparse signals in basis \( \Psi \); and \( M = \{ u \in \mathbb{R}^n : |Fu| = y \} \), where \( F \) denotes the \( n \times n \) discrete Fourier transform (DFT) matrix, be the set of all signals whose DFT magnitude matches the measurement vector \( y \). In sparse PR, we seek to find a signal \( \hat{x} \) such that \( \hat{x} \in S_0 \cap M \). We recently proposed two iterative fixed-point algorithms [1], referred to as max-\( s \) and relaxed averaged alternating reflections with \( \ell_0 \) constraint (\( \ell_0 \)-RAAR), for solving this non-convex feasibility problem. Their update rules are given by \( x^{t+1} = \mathcal{P}_{S_0} \left( \mathcal{P}_M x^t \right) \) and \( x^{t+1} = T_\beta (x^t) \), respectively, where \( T_\beta \) is a combination of appropriately defined projection and reflection operators [2]. The max-\( s \) algorithm has the error reduction property, meaning that the measurement domain reconstruction error does not increase with iterations. The \( \ell_0 \)-RAAR algorithm offers a generalized parameterized framework for FDOCT reconstruction, of which max-\( s \) and the direct inversion technique turn out to be special cases, corresponding to \( \beta = \frac{1}{2} \) and \( \beta = 0 \), respectively. The performance of max-\( s \) and \( \ell_0 \)-RAAR is validated on a specimen formed by stacking two microscopic glass slides. The sparsity is chosen as \( s = 0.05n \), where \( n = 2048 \) samples are taken along each scan-line. The sparsifying basis \( \Psi \) is chosen to be the orthonormal Haar basis. We observe (cf. Fig. 1) that the proposed techniques yield tomograms without autocorrelation artifacts and superior signal-to-background noise ratio compared with the state-of-the-art.

References