ABBE THEORY AND THREE-DIMENSIONAL SPATIAL BANDWIDTH

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The Abbe theory specifies the spatial cutoff frequency that limits a spatial bandwidth of microscope optics in a lateral direction [1]. In general microscope's optics is a three-dimensional (3D) spatial filter and it should be characterized by the 3D optical transfer function [2].

It is possible to get the analytical description of the 3D surface that "covers" 3D space of spatial harmonics that can pass through a microscope objective [3]. This space is called the 3D spatial bandwidth. To get this description it has to consider a grating that can be inclined relatively the optical axis. Then it has to calculate the maximal path difference of beams that form 0- and ±1- maximums as the Abbe theory does. As a result the proposed analytical expression links the spatial cutoff frequency with a wavelength, a refractive index, an angle of a grating inclination and an aperture angle [3]:

$$\nu_{MAX} = \nu_{MAX}(\sigma, \alpha) = \frac{1}{d_{MIN}(\sigma, \alpha)} = \frac{n \cdot \max\left(\left|\sin(\lim(\alpha \pm \sigma)) - \sin(\lim(\alpha \mp \sigma))\right|\right)}{\lambda}$$

where ν_{MAX} , d_{MIN} – the maximal spatial frequency passed through a microscope objective (the spatial cutoff frequency) and the corresponded minimal resolvable period of the grating when the numerical apertures of an objective and a condenser are equal, relatively; α – the angle of the grating inclination $-\pi/2 \le \alpha \le \pi/2$; σ , λ, n – the aperture angle of the microscope objective $\sigma < \pi/2$, the wavelength and the refractive index in an object space, relatively; max – the function that returns the maximal its argument; lim – the function that sets the limits of its argument:

$$\lim(\varphi) = \begin{cases} \pi/2, \varphi \ge \pi/2 \\ \varphi, -\pi/2 < \varphi < \pi/2 \\ -\pi/2, \varphi \le -\pi/2 \end{cases}$$

The known formulas that define the spatial cutoff frequencies in the lateral and axial directions can be considered as the partial cases of the proposed formula:

$$\nu_{MAX}(\sigma, \alpha = 0) = \frac{n \cdot \max\left(\left|\sin(\lim(\sigma)) - \sin(\lim(-\sigma))\right|\right)}{\lambda} = \frac{2 \cdot n \cdot \sin(\sigma)}{\lambda}$$

$$\nu_{MAX}\left(\sigma, \alpha = \frac{\pi}{2}\right) = \frac{n \cdot \max\left(\left|\sin\left(\lim\left(\frac{\pi}{2} \pm \sigma\right)\right) - \sin\left(\lim\left(\frac{\pi}{2} \mp \sigma\right)\right)\right|\right)}{\lambda} = \frac{n \cdot (1 - \cos(\sigma))}{\lambda}$$

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