TRANSFER FUNCTION ANALYSIS OF AXIAL DERIVATIVES IN TRANSPORT-OF-INTENSITY QUANTITATIVE PHASE IMAGING

Zhengyun Zhang\textsuperscript{1}, George Barbastathis\textsuperscript{1,2}

\textsuperscript{1}Singapore-MIT Alliance for Research and Technology, BioSyM IRG
77 Massachusetts Avenue, Cambridge, MA 02139, USA
\textsuperscript{2}Massachusetts Institute of Technology, Department of Mechanical Engineering
1 CREATE Way, #04-13/14 Enterprise Wing, Singapore 138602, Singapore

E-mail : zhengyun@smart.mit.edu

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Transport-of-intensity equation (TIE) quantitative phase imaging methods extract a phase image from the axial derivative of transverse intensity \cite{1}. However, this derivative is usually estimated from two or more transverse intensity images. These finite difference approaches have been studied extensively in previous literature, e.g. in Refs. 1–4. We propose an alternative, yet intuitive way of analyzing any finite difference approach by modeling the derivative estimate \( d(x,y) \) as a slice of the convolution of the 3D intensity distribution \( I(x,y,z) \) with an approach-specific kernel \( h(z) \). Thus, each spectral component \( \tilde{d}(fx, fy) \) of the estimate is the inner product of the transfer function \( \tilde{h}(fz) \) and a slice \( s(fz) = \tilde{I}(fx, fy, fz) \) of the intensity’s Fourier transform. This analysis shows that a standard two-plane approach results in a sinusoidal transfer function approximating the derivative operator \( j2\pi fz \) near the origin, but if the \( \Delta z \) separation becomes too large, the well-known breakdown of the estimate can be seen as the sinusoid becoming nonlinear within the doughnut-shaped support of \( \tilde{I}(fx, fy, fz) \). It also shows the middle transverse spatial frequencies (i.e. the thickest part of the doughnut) having the highest error, instead of the highest spatial frequencies; numerical simulations of a bead corroborate this result. We expect this analysis technique to not only provide intuitive explanations of previous observations but also yield new insights for transport-of-intensity equation quantitative phase imaging.

![Diagram](attachment:image.png)

Figure 1: The transfer function \( \tilde{h}(fz) \) of a two-plane finite difference estimator is shown in (a); only the linear regime (solid) near the origin is a good estimate of the derivative operator. The support of \( \tilde{I}(fx, fy, fz) \) is shown in gray in (b), with the hatched region (i.e. where the doughnut is thickest) highlighting the middle transverse frequencies where derivative estimation error should be large. When imaging a sub-resolution bead illuminated by a plane wave, the RMS error as a function of \( fr = \sqrt{fx^2 + fy^2} \) for the derivative estimate (c) and the recovered phase (d) averaged across 100 simulations with noise show similar features, albeit skewed due to nonuniformity in \( \tilde{I} \).


