

# ERNST ABBE AND THE ORIGINS OF FOURIER OPTICS

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## 1. INTRODUCTION

According to the Abbe theory of image formation [1], if a grating object is illuminated coherently and observed using a microscope objective, the diffraction orders of the grating are observed in the back focal plane of the objective [2, 3]. These grating orders are then combined to form an image in the image plane. This is the basic description of image formation according to Fourier optics: the objective produces a Fourier transform of the object in the back focal plane of the lens, and propagation to the image plane produces another Fourier transformation, equivalent to an inverse Fourier transform [4]. The resolution of the image depends on the number of grating orders that are transmitted through the optical system. It was later shown that, if these grating orders are modified, different imaging modes such as dark field, Zernike phase contrast or Schlieren imaging can be generated.

## 2. COHERENT AND INCOHERENT IMAGING

However, in a microscope the object is not usually illuminated coherently. The illumination system of a brightfield microscope can be critical illumination or Köhler illumination. In Abbe's time it was not clear how these differed, and much effort was expended on achieving perfect critical illumination, but now it is known that in principle they behave identically [5], although there are some practical advantages of Köhler illumination. In 1872, Rayleigh [6] proposed his resolution criterion, which applies for incoherent imaging. There was thus difficulty in reconciling these two theories of imaging, which is sometimes presented as a dispute between Abbe and Rayleigh. In fact there is no dispute because the two theories are concerned with coherent and incoherent imaging, respectively.

## 3. PARTIALLY COHERENT IMAGING

The Fourier theory of image formation with partially coherent illumination was eventually published by Hopkins [7]. This theory shows that under either critical or Köhler illumination, the degree of partial coherence is determined by the numerical aperture of the condenser, relative to that of the objective. Thus coherent and incoherent image formation correspond to two limiting cases, for small and large relative condenser apertures, respectively.

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